Number Strings: Developing Computational Efficiency in a Constructivist Classroom

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Children develop stronger number sense if allowed to explore strategies when computing, instead of being tied down to rigid procedures, such as algorithms. Guided by this belief, DiBrienza and Shevell demonstrate the powerful role of number strings in promoting computational efficiency.

One of the main criticisms and concerns parents and administrators have about constructivist mathematics teaching is that children do not learn to compute quickly and accurately. Because of this misperception, teachers are under pressure to teach standard algorithms. Much of Constance Kamii's work shows that this approach is misguided and may, in fact, be detrimental to children's understandings about number (Kamii, 1998).

Children should be allowed to come up with their own strategies for computation. Yet in many instances, the strategies that children invent, while they make sense to the student and may be mathematically correct, are cumbersome and inefficient. What can teachers do to be sure their students are developing a repertoire of efficient computation strategies when working with numbers? As educators, we must provide opportunities for children to hone and develop big ideas and to strive for efficiency and elegance in their strategies. One such opportunity involves mental-math activities using *number strings.*

What Is a Number String?

A number string is a series of related but bare (devoid of context) computation problems that are specifically designed to elicit quick, efficient, and reliable strategies for computation from students. The problems are written horizontally, not vertically. When problems are written this way, they do not encourage any one particular strategy. Writing problems vertically, on the other hand, inherently suggests a place-value splitting strategy.

Number strings give students a chance to notice patterns and hone their computational skills in a constructivist way. The goal is for children not to be bound to a rigid procedure such as an algorithm that is used regardless of the problem, but rather, to look to the numbers to decide which strategy to use. After all, this is what mathematicians do; they look for and create elegant solutions.

Take the following problem:

4,017 – 3,998.

When students solve this using the traditional algorithm, they must "borrow" a 1 from the tens column because 8 is larger than 7. Next, they must "borrow" a 1 from the hundreds column; however, that is not possible, so they must change the 4 from the thousands column into a 3, give the extra thousand to the hundreds column, change the hundreds column from a 10 to a 9, give the extra hundred to the tens column and finally subtract each column. Then—and only if the student has done every step correctly—will she have the correct answer. Moreover, if she made a mistake, the student has no way to check except to repeat the algorithm or perform an addition algorithm to check her subtraction algorithm. There is no use of number sense.

A child from a constructivist classroom who has been allowed to invent procedures might say, "4,000 take away 3,000 is 1,000. 1,000 take away 900 is 100. 100 take away 90 is 10. 10 take away 8 is 2, plus 17 is 19, so the answer is 19." (See Figure 1.)

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  4,017
- 3,998
-------
  1,000
-   900
-------
    100
-    90
-------
     10
-     8
-------
      2
+    17
-------
     19
```

Figure 1.

The student first looked to the numbers to decide which strategy would work best and then proceeded. This child has some number sense, but is this how we want children to solve problems like this?

A child from a constructivist classroom who has consistently worked with number strings might use a constant-difference strategy: add 2 to both numbers, making the problem

4,019 – 4,000.

This answer also is 19. This child has exhibited true number sense. He did not jump right in and perform an algorithm, whether standard or invented. Nor did he perform an invented strategy that is inefficient for these numbers. Rather, he first looked to the numbers to decide which strategy would work best and then proceeded. It is clear that this child has some deep understandings about subtraction. He is able to treat the numbers as whole, without breaking them into unnecessary parts. Further, he understands the value of landmark numbers and is capable of using them to make this problem friendlier. Finally, he understands subtraction as difference and that for these numbers, adding two to each, maintaining the difference, is a more efficient strategy for subtracting than removing numbers.

All of these ideas can be explored using number strings. Figure 2 is an example of a string that is likely to elicit a constant-difference strategy.

150 – 75
151 – 76
149 – 74

294 – 100
291 – 97
301 – 107

Figure 2.

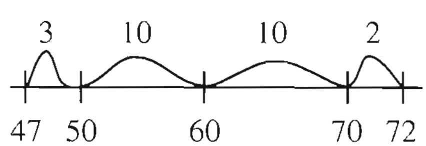
Children see that the first three answers are the same and then investigate how they are related. In the next three problems, the situation repeats itself, giving students an opportunity to explore further. Eventually, when they understand the strategy, they add it to their repertoire.

When doing a string with a group of students, the teacher places a single problem on the board horizontally. The teacher then gives the students think time to mentally solve the problem and prepare to verbalize what they did. As children share their ideas, the facilitator visually represents what they say. Students hear and see representations of their peers' strategies, and they can discuss the variety of approaches.

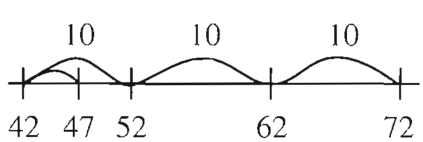
For example, the teacher may write this problem on the board:

72 – 25.

One student might say, "First I took 2 away from the 72 to get to 70. Then I jumped back 10 to 60. Then I jumped back another 10 to 50. I still had 3 more to take away. 50 minus 3 is 47." The teacher may draw on the board:



Another student may add, "I did it differently. I started at 72, jumped back three 10s to 42, and then added 5 back on, so the answer is 47." The teacher draws on the board:



After a few different strategies are shared, discussed, and clarified, the next problem in the string is introduced with the same procedure. This process continues throughout the string, while the facilitator capitalizes on opportunities to further student thinking.

To be effective, number strings need to be explored on a consistent basis. It takes time and exploration to construct a deep understanding of any one strategy, and there are many strategies worth exploring. In order for students to truly look to the numbers, they must explore all of these strategies in conjunction with one another. If they are only able to explore certain big ideas, the related strategies risk becoming algorithms themselves.

Only by doing string work regularly can students develop efficiency in their computation. *However, number strings are not a substitute for hands-on investigation and exploration.* Strings can cause students to raise questions around number, but children will not construct the mathematical big ideas embedded in number strings unless these properties of number are also investigated in hands-on, child-directed mathematical investigation.

Addition and Subtraction Strings

The following are some of the *addition and subtraction* big ideas that our students have constructed and explored through string work.

• *Keeping the first number whole and adding or subtracting by moving to the nearest 10 (or 100) or making jumps of 10 (or 100)*
Through various investigations and games, students begin to recognize the importance of 10 as a landmark number. Figures 3 and 4 provide examples of strings that can be explored in conjunction with this concept.

• *Counting up to subtract*
This is another important big idea for subtraction. After students discover, through context problems, games, and investigations, that it is possible to count up when subtracting, as well as to count backwards, the teacher can use strings to explore when it might make more sense to count up than to remove. Figure 5 is an example of a string that is designed to explore this idea.

Children working on this string might realize that for some of these problems, counting up to subtract is quite a task, while for the others, it is rather easy. The teacher can then facilitate a class conversation around why certain numbers beg for particular strategies.

Multiplication and Division Strings

The following are some of the big ideas through which we can investigate *multiplication and division.*

• *Doubling and halving*
As students work with arrays to explore multiplication, they begin to discover that by rearranging the array, the problem can be changed, yielding the same product. For example, if students are presented with 3 1/2 x 14,

Moving to the Nearest 10	
17 + 3	42 – 2
17 + 6	42 – 7
27 + 3	62 – 2
27 + 16	62 – 27
57 + 3	72 – 2
57 + 36	72 – 37

Figure 3.

Making Jumps of 10	
27 + 10	33 – 10
37 + 10	53 – 10
47 + 20	53 – 20
47 + 24	53 – 24
56 + 30	83 – 50
56 + 35	83 – 54

Figure 4.

101 – 97
101 – 6
153 – 145
153 – 14
513 – 489
513 – 24
1,003 – 992
1,003 – 27
4,017 – 3,998
4,017 – 78

Figure 5.

they can solve it by doubling the 3 1/2 and halving the 14, making the problem 7 x 7 (see Figure 6).

With continued string work, we see that the strategy can go beyond doubling and halving, as shown in Figure 7.

Here is a division example: 300/12.

If both numbers are cut into thirds, the problem becomes 100/4, as in Figure 8.

Figure 9 shows a division string that might bring up this strategy for investigation.

• *Using landmark numbers and using landmark numbers with compensation*

The distributive property of multiplication is another big idea around which we can explore multiplication and division. For example, the arrays in Figure 10 show how the distributive property can be used for double-digit multiplication.

Figure 11 shows multiplication and division strings that explore this idea.

The Role of Conversation

Student interaction and conversation around the strategies they use are crucial aspects of string work. During conversation, students are held accountable to try to make sense of each other's strategies and defend their own. Accountable talk seriously responds to and further develops what others in the group have said (Institute for Learning, LRDC,

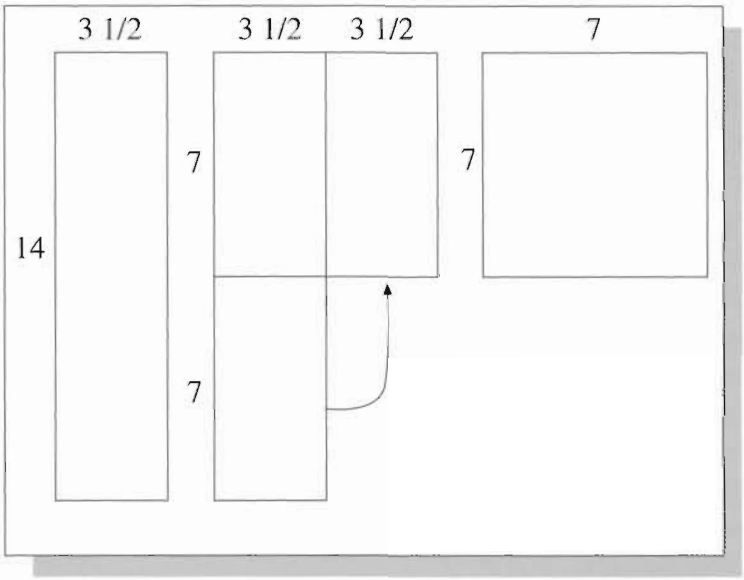


Figure 6.

3 x 6
3 x 12
6 x 12
3 x 24
9 x 8
6 x 24
12 x 12
4 x 36
2 x 18

Figure 7.

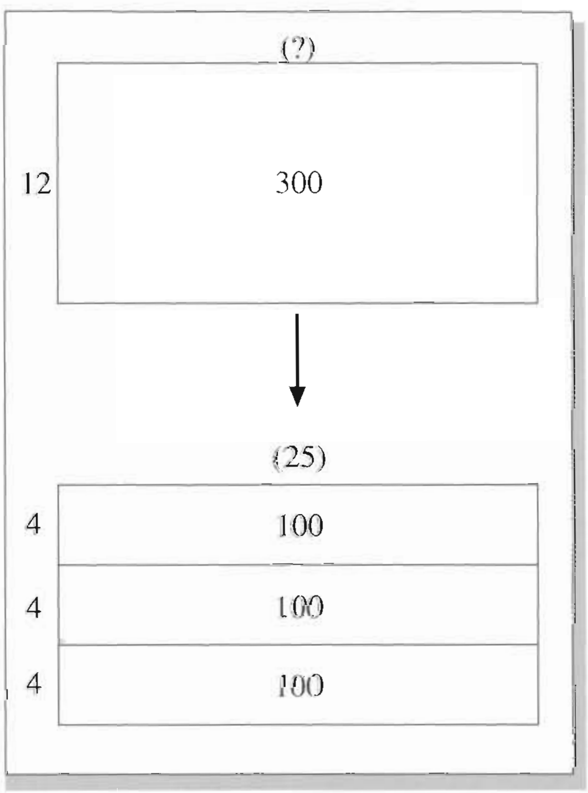


Figure 8.

16 / 4
32 / 4
64 / 8
32 / 8
32 / 16
64 / 16
128 / 32

Figure 9.

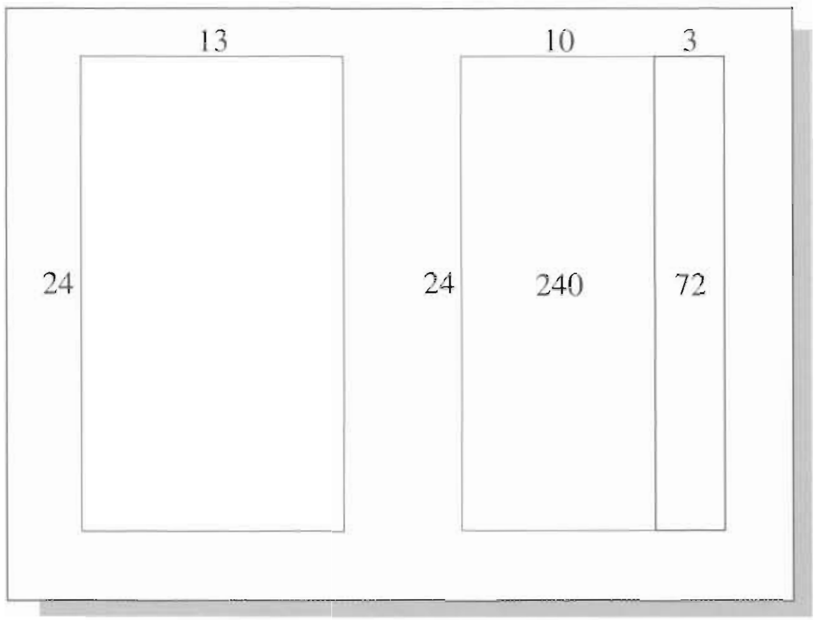


Figure 10.

10 x 10	100 / 4
1 x 10	36 / 4
11 x 10	136 / 4
11 x 3	144 / 12
11 x 13	48 / 12
12 x 10	192 / 12
12 x 14	143 / 13

Figure 11.

1997). By sharing their strategies, students are afforded opportunities to further their own understandings. String conversations may also provide opportunities to try other strategies, to explore why an approach works, and to debate about efficiency.

Conclusion

Mental math using number strings is a powerful way to focus conversation on computation strategies and to develop the eventual goal of number sense—number sense that is strong enough to enable students to look to the numbers before they decide on their strategy.

We need to teach math, of course, primarily through learner investigations, but mental math can be a powerful mini-lesson at the start of math workshop. Also, whenever a teacher has 15 minutes, she can use a number string. Morning meetings and/or transitions are excellent opportunities as well.

The more students work with strings, the more efficient their strategies will be. Children who are given opportunities to explore and construct strategies will derive aesthetic pleasure in playing with numbers and searching for elegant solutions. ❐

References

Kamii, C. (1998, April). *The harmful effects of algorithms.* Paper presented at the annual meeting of the National Council on Teaching of Mathematics. Washington, DC.

Institute for Learning, LRDC. (1997). *Instruction and learning profile.* Pittsburgh, PA: University of Pittsburgh.

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